

SHAPE AND TIME EVOLUTION OF THE GAS-LIQUID INTERFACE IN THE PRESENCE OF AN EXTERNAL PERTURBATION

Karol Wędołowski

An interface is an idealized two-dimensional representation of the boundary region separating two different kinds of matter or two phases of the same matter. A special class of interfaces consists of those which separate two liquids or liquid and gas. Deformation of the interfacial shape introduces additional degrees of freedom to the system, which give rise to many interesting phenomena but also to new technical difficulties in their theoretical analysis.

The equilibrium shape of an interface can, on the macroscopic level, be found using the Laplace equation. However, this description might turn insufficient when characteristic distances between interfaces are small enough, usually below a micrometer. In that case, additional effects related to a finite range of molecular interactions need to be taken into account. By integrating microscopic degrees of freedom in the system, one may obtain the effective interaction potentials between the interfaces expressed as functions of their geometrical configurations. As a result, an effective hamiltonian for the interface can be derived, which in turn leads to the generalized Laplace equation. This equation includes additional terms - disjoining pressures - which reflect the effective interaction between the interfaces.

When a fluid is placed in the vicinity of a wall or a substrate effective interactions generated by the dispersive van der Waals forces affects its surface properties significantly, e.g. playing a key role in the wetting phase transitions. However, there are many systems where the gas - liquid interface experiences, aside from the effective interaction with a substrate, an effective interaction with a localized perturbation. A perturbing object can be, for example, a droplet or a bubble, a colloidal particle or a tip of an atomic force microscope. The localized perturbation induces deformation of the interface which may extend to distances of the order of capillary length - potentially far beyond the size of the perturbing object. Effective interaction between the gas-liquid interface and the surface of the perturbing object generated by the van der Waals forces is usually attractive. It can cause a capillary bridge formation when the distance between the object and the interface is small enough.

Equilibrium analysis of this problem requires an assumption that the time scales of the interface deformation are short comparing to the time scale of the experiment so that the dynamic effects related to the relaxation processes can be neglected. However, it appears that this condition is not satisfied in many systems and the relaxation process can be significantly longer than duration of the experiment. Generalized Laplace equation can be utilized in the hydrodynamic boundary condition for pressure at the gas-liquid interface, which couples the effective description of the interactions between interfaces with the Navier-Stokes equations for the liquid film dynamics. However, this approach is computationally demanding when used without additional simplifications. For this reason, many analysis make use of the thin liquid film approximation, which transforms the Navier-Stokes equations into a single equation for the thickness of the liquid layer. However, this approximation requires that characteristic longitudinal length scales in the system are much larger than the film thickness, which may not be true for many experimental setups.

In this dissertation, an alternative approach to analysis of the gas-liquid interface deformation is proposed. One of its main outcomes is the derivation (directly from the Navier-Stokes equations) and analysis of a general dispersion relation for free surface perturbations on a viscous liquid film of arbitrary thickness. The general dispersion relation simplifies, in the limit of large Reynolds numbers, to known from literature forms of dispersion relations for the perfect (inviscid) liquid and exhibits interesting properties in the range of parameters, for which viscosity plays a dominant role in liquid film dynamics. For example, the real part of the frequency has a non-zero value only in a finite range of wave numbers, and exhibits non-analytical behavior in the transition regions.

The general dispersion relation and its asymptotic forms have been used to analyze deformation

of the gas-liquid interface caused by the interaction between the liquid film and a solid ball located above its free surface. In the approximation of low Reynolds numbers an evolution equation for the deformation spectrum has been derived. After its linearization characteristic deformation time scales have been obtained and related to the length scales present in the system. Also, numerical methods have been proposed for analysis of the nonlinear problem, for which the disjoining pressure depends on the deformation of the interface. They have been used to find the time evolution of the interface in case the ball moves vertically (e.g. oscillating) above the liquid film.

Aside from the axisymmetric configurations, the interface deformation has been analyzed for the ball moving along the gas-liquid interface with a constant velocity. In this problem an important parameter is the wave resistance, which is a horizontal drag force experienced by the perturbing object moving in the vicinity of the interface. The wave resistance has been calculated using the general dispersion relation as well as its asymptotic forms in the regimes of small and large Reynolds numbers. Special attention has been paid to the role of the film thickness and the velocity of the perturbation in the dynamics of the system. It has been shown that there is a wide range of the system parameters in which an accurate description of the interface requires using the full form of the general dispersion relation.